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Long-Term Population Projections under Uncertainty

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Independent Authority for Fiscal Responsibility

AIReF Autoridad Independiente de Responsabilidad Fiscal

DISCLAIMER

- This presentation is based on joint work with Ainhoa Osés-Arranz: "Introducing Uncertainty on Fertility and Survival in the Spanish Population Projections: A Monte Carlo Approach", AIReF Working Paper DT/2018/5.

- Any views expressed herein are those of the author and not necessarily those of the Spanish Independent Authority for Fiscal Responsibility (AIReF).

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- Motivation.
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- Introducing uncertainty:
  - **Modeling fertility rates.**
  - Survival rates.
  - Migration.
- Final recap.
- The road ahead.

MOTIVATION

- **WHY?**

- Population projections are a key input for:
  - Long-term growth forecasts.
  - Expenditure projections: pensions, health, education, etc.
  - Economic policy: labor market, fertility, migration, etc.

- As a result, full control of the projections (including a measure of its uncertainty) was deemed essential for the proper assessment of fiscal policy and long-term sustainability.

- In the end, this practice mimics the one followed with economic projections: assessment by means of a model-based approach.

GENERAL FRAMEWORK

- **HOW?**
- As in other fields, the basic idea is fairly simple (some may say it is even trivial!) but a state of the art implementation is hampered by a bunch of technicalities. As usual, the devil is in the details.
- We have used a modular approach to subdue the complexities and to have an operative implementation, including a well-designed software code.
- Let’s start!

GENERAL FRAMEWORK

- The basic idea underlying cohort dynamics is an accounting identity that relates STOCKS and flows:

  - PEOPLE TODAY = (PEOPLE YESTERDAY – deceased)
  - \+ newborn + net migration =
  - = SURVIVORS + newborn + net migration

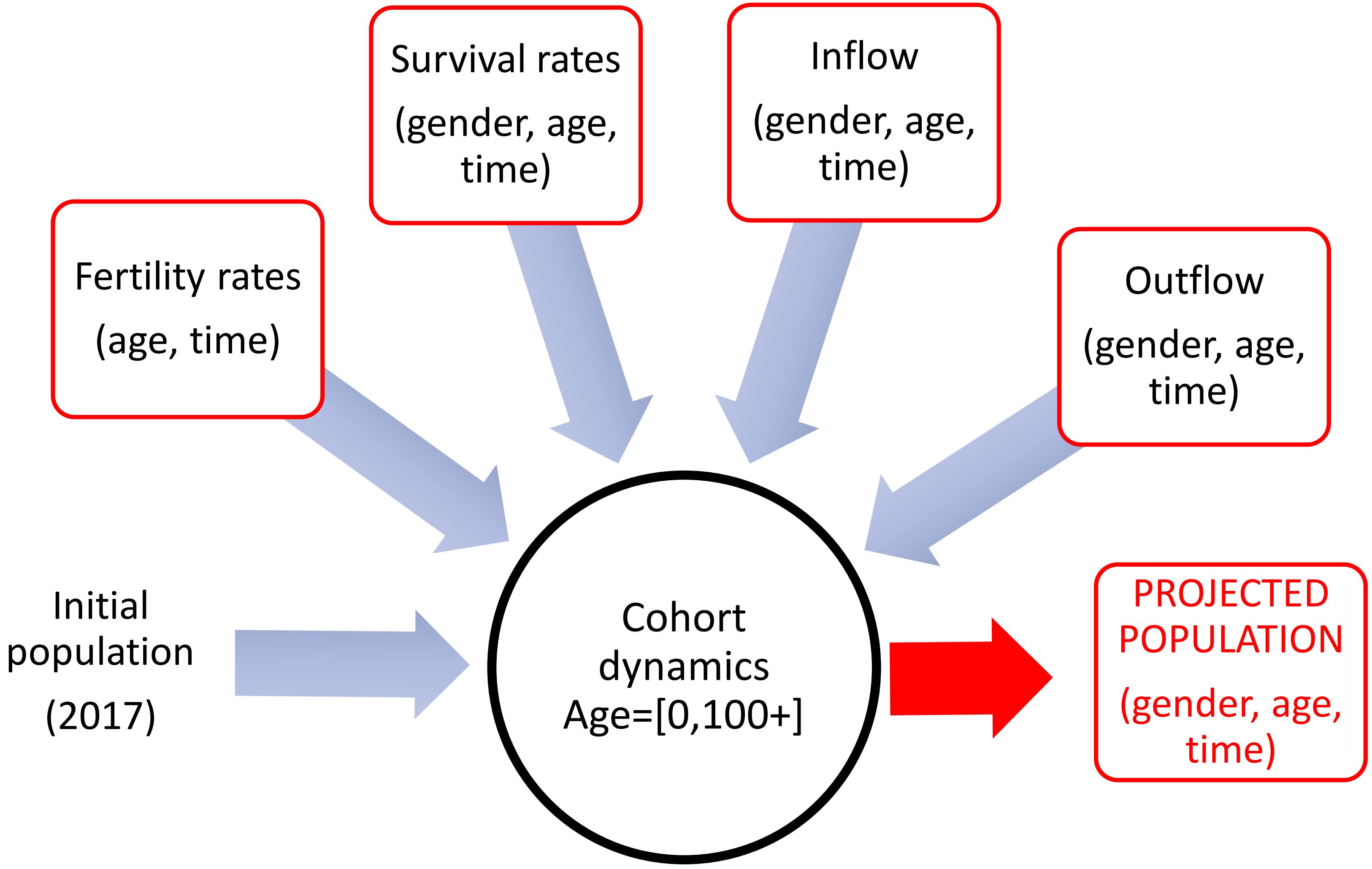
  - Modeling approach:

  - SURIVORS $\propto$ PEOPLE YESTERDAY = (survival probability) x (PEOPLE YESTERDAY)
  - newborns $\propto$ PEOPLE YESTERDAY = (fertility rates) x (PEOPLE YESTERDAY)
  - net migration $\propto$ PEOPLE (here and abroad) YESTERDAY

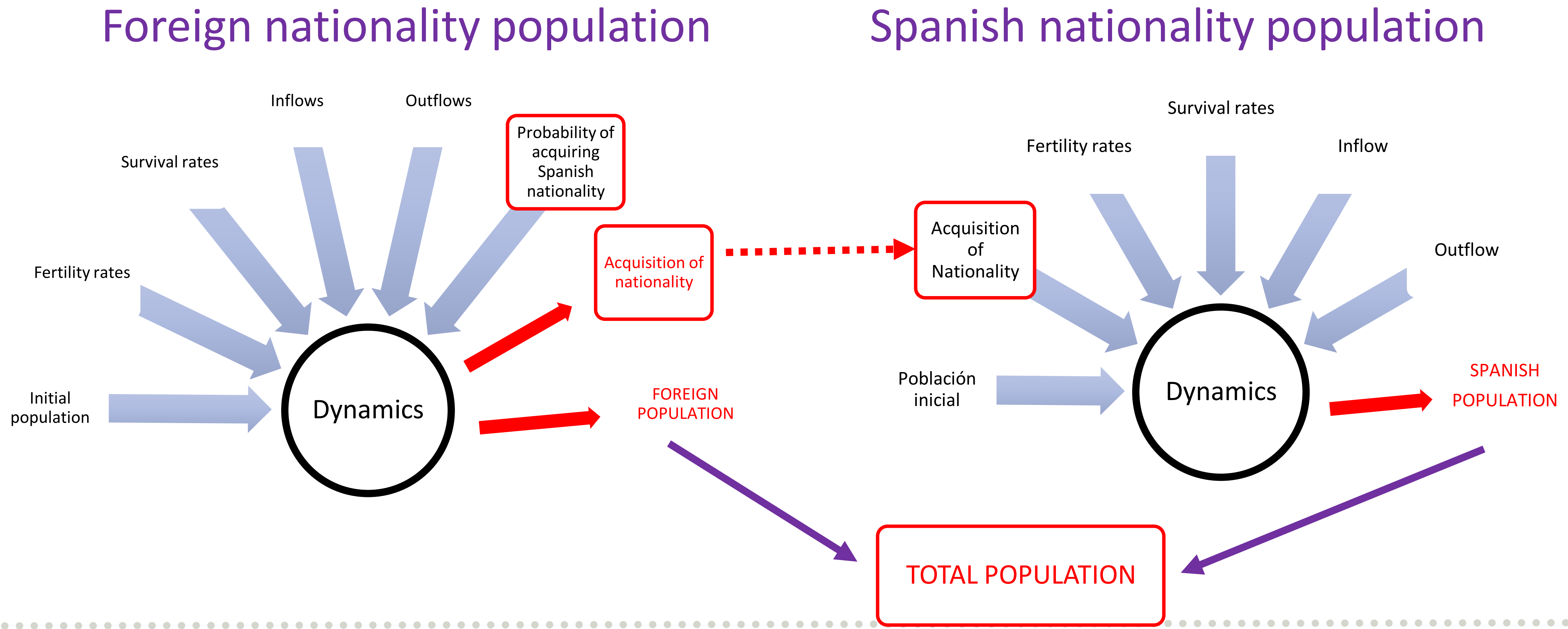
GENERAL FRAMEWORK

- Population projections are based on the **cohort method**, with a breakdown by age (from 0 to 100+), gender and nationality (foreign or Spanish).
- The main inputs of the model (fertility, survival and net migration) are represented using dynamic stochastic models in order to quantify the uncertainty around the forecasts.
- In addition, fertility and survival are modeled using a two-step approach that combines domestic and international information.
- Net migration is modeled by means of a multilateral gravity model. Its inputs are the population and the economic dynamics of a set of countries that form a bilateral grid of senders and receivers of migration flows.

GENERAL FRAMEWORK



GENERAL FRAMEWORK: nationality breakdown



GENERAL FRAMEWORK: nationality breakdown

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- Inertial component linked to fertility rates and survival rates

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$$

- Exogenous migration: inflows and outflows
- Acquisition of nationality: positive if v=es (Spanish) and negative if v=f (foreign)

INTRODUCING UNCERTAINTY: Fertility

- Fertility rates are projected using explicitly stochastic dynamic models.

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INTRODUCING UNCERTAINTY: Survival

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- Survival rates are projected using explicitly stochastic dynamic models.

INTRODUCING UNCERTAINTY: Migration

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- **Gravity models** are used to explain and project migration, see Osés-Arranz (2019).

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$$

MODELING FERTILITY (AND SURVIVAL)

- **First step:**
  - For each year, we estimate a parametric model for the age-specific fertility rates and survival rates (men and women separately).
  - The chosen functional forms are:

$$f_{i,t} = \beta_{0,t} \exp\left[-\left(\frac{i-\beta_{1,t}}{\beta_{2,t}}\right)^2\right] + e_{i,t}$$

Intensity

Modal age

Dispersion

$$s_{i,t} = \beta_{0,t} + \beta_{1,t} i^{\beta_{2,t}} + e_{i,t}$$

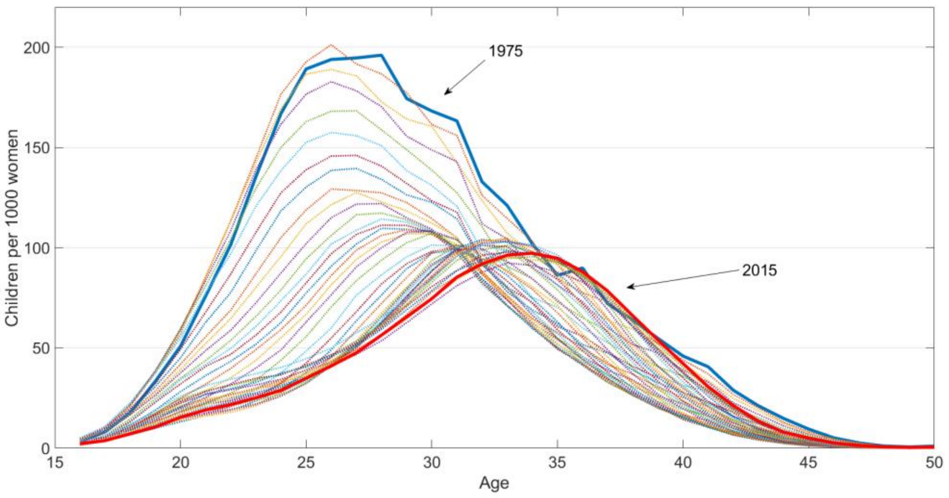
Benchmark

Intensity

Curvature

- Ease of interpretation and data fit were the main selection criteria.

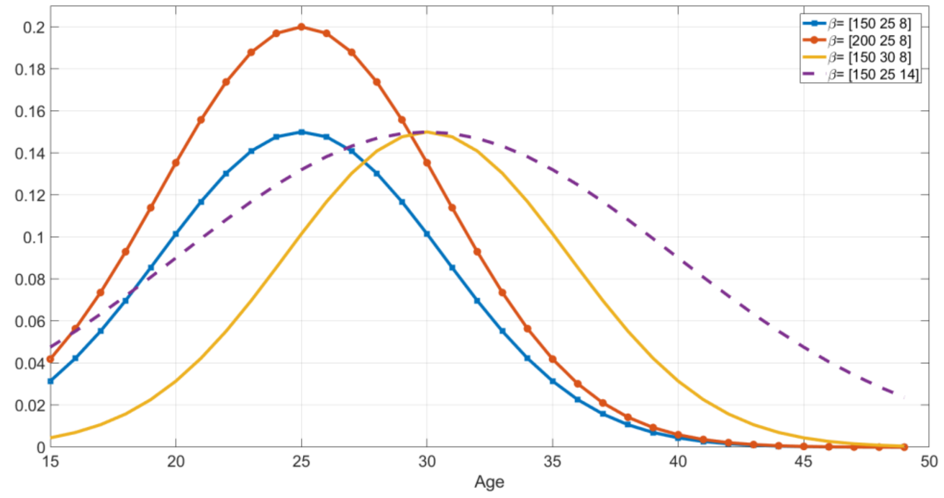
MODELING FERTILITY: First step

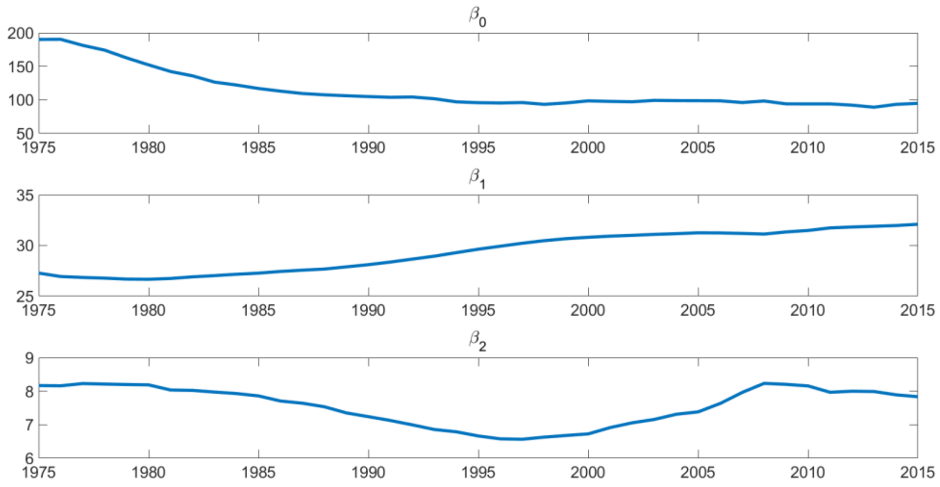


For each year we fit a parametric curve:

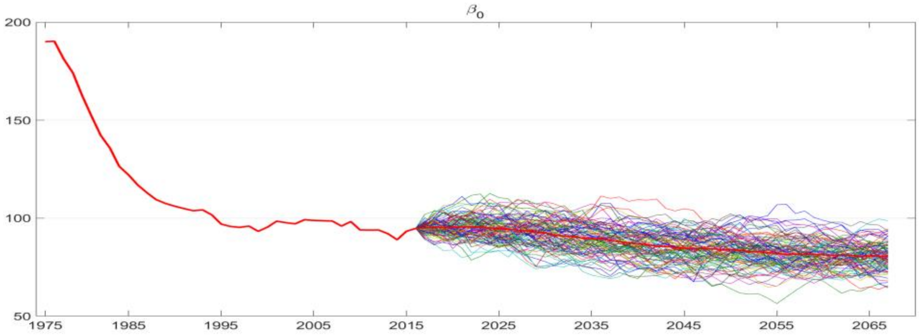
$$f_{i,t} = \beta_{0,t} \exp\left(-\left(\frac{i - \beta_{1,t}}{\beta_{2,t}}\right)^2\right) + e_{i,t}$$

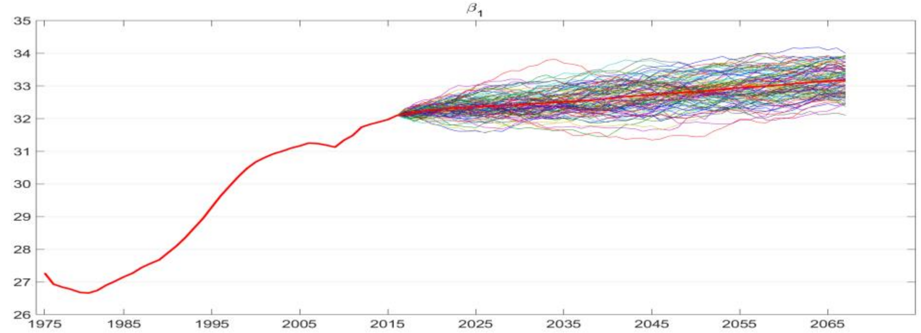
The time series of the three parameters are modeled using a VAR model. This model is used to generate multiple (and consistent) stochastic scenarios by means of Monte Carlo simulation.

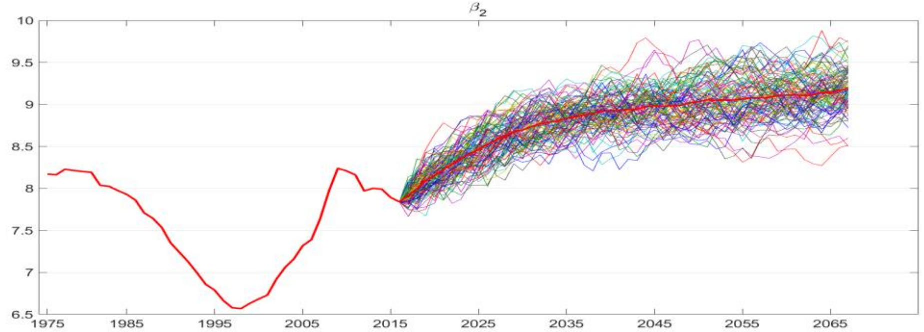


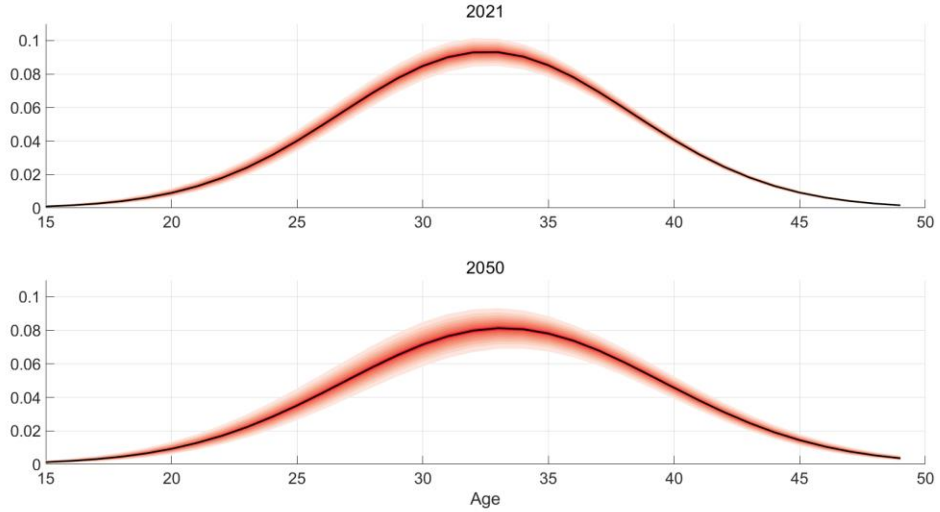


MODELING FERTILITY: First step





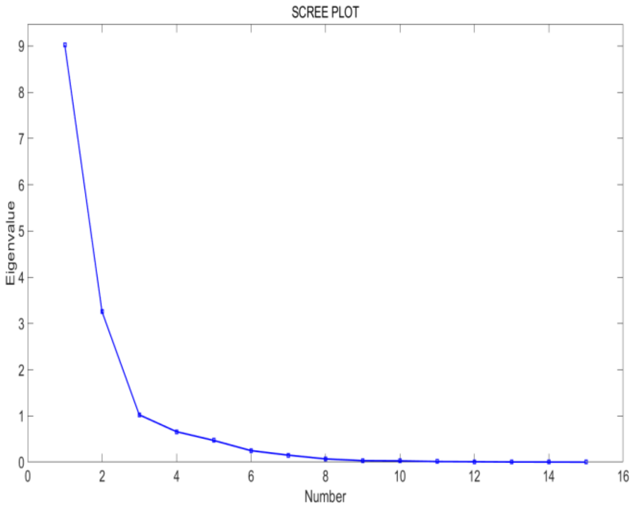


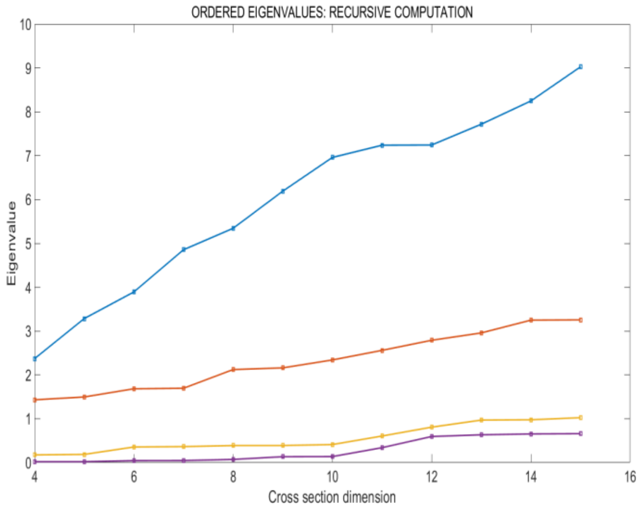


MODELING FERTILITY (AND SURVIVAL)

- **Second step**: we condition the long-term evolution (2100) of the β parameters to a benchmark derived from a factor model.
- The factor model represents the joint dynamics of (aggregate) fertility and survival of a panel of European countries.
- The factor model estimates a high degree of commonality within the panel, justifying the generation of a benchmark that operates as an attractor for the individual countries.
- The linkage between the long-term benchmark and the stochastic projections is made through the intensisty parameters of both fertility and survival.
- The technical device is a simplified Brownian bridge (a stochastic process with given initial and terminal conditions).

MODELING FERTILITY: Second step





$$y_{j,t} = \lambda_{j,1} z_{1,t} + \lambda_{j,2} z_{2,t} + \cdots + \lambda_{j,r} z_{r,t} + e_{j,t} \qquad j = 1 \ldots M \quad t = 1 \ldots n$$

- $y_{j,t}$: Number of children per women (or life expectancy at birth) of country *j* at time *t*.
- $z_{h,t}$: Common factor *h* at time *t*, where $h = 1..r$ being $r < M$.
- $\lambda_{j,h}$: Loading of factor *h* on country *j*.
- $e_{j,t}$: Idiosyncratic term of country *j* at time *t*.

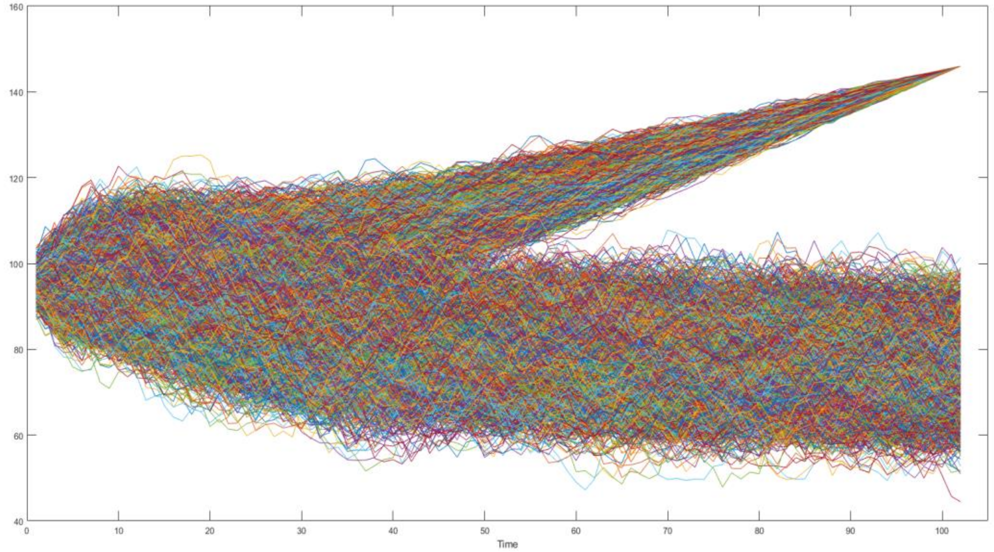
	Loadings		Communalities			Scores	
Country	1f	2f	1f	2f	All factors	1f	2f
Austria	0.95	-0.17	0.90	0.03	0.93	0.11	0.05
Belgium	0.79	0.51	0.63	0.26	0.89	0.12	0.27
Denmark	0.46	0.69	0.21	0.47	0.68	0.04	0.10
Finland	-0.48	0.60	0.23	0.36	0.59	0.00	0.05
France	0.85	0.32	0.72	0.10	0.82	0.07	0.12
Greece	0.80	-0.54	0.64	0.29	0.93	0.04	-0.19
Ireland	0.88	-0.46	0.77	0.21	0.98	0.19	-0.44
Italy	0.96	-0.20	0.92	0.04	0.96	0.18	0.05
Netherlands	0.72	0.58	0.52	0.33	0.85	0.09	0.20
Norway	0.77	0.52	0.60	0.27	0.86	0.10	0.21
Portugal	0.84	-0.48	0.71	0.23	0.94	0.06	-0.17
Spain	0.88	-0.43	0.77	0.18	0.96	0.10	-0.18
Sweden	0.20	0.59	0.04	0.35	0.40	0.01	0.04
Switzerland	0.89	0.08	0.80	0.01	0.81	0.05	0.06
UK	0.75	0.34	0.57	0.11	0.68	0.03	0.07
Median	0.80	0.48	0.64	0.23	0.86	0.07	0.05
MAD	0.25	0.14					

MODELING FERTILITY: Second step

Country	Scores	Weights	UN 2095-2100
Austria	0.11	0.09	1.79
Belgium	0.12	0.10	1.87
Denmark	0.04	0.03	1.86
Finland	0.00	0.00	1.83
France	0.07	0.06	1.94
Greece	0.04	0.03	1.76
Ireland	0.19	0.16	1.91
Italy	0.18	0.15	1.79
Netherlands	0.09	0.07	1.83
Norway	0.10	0.08	1.86
Portugal	0.06	0.05	1.76
Spain	0.10	0.09	1.72
Sweden	0.01	0.01	1.93
Switzerland	0.05	0.04	1.70
UK	0.03	0.03	1.86
Sum / Weighted average		1.00	1.83

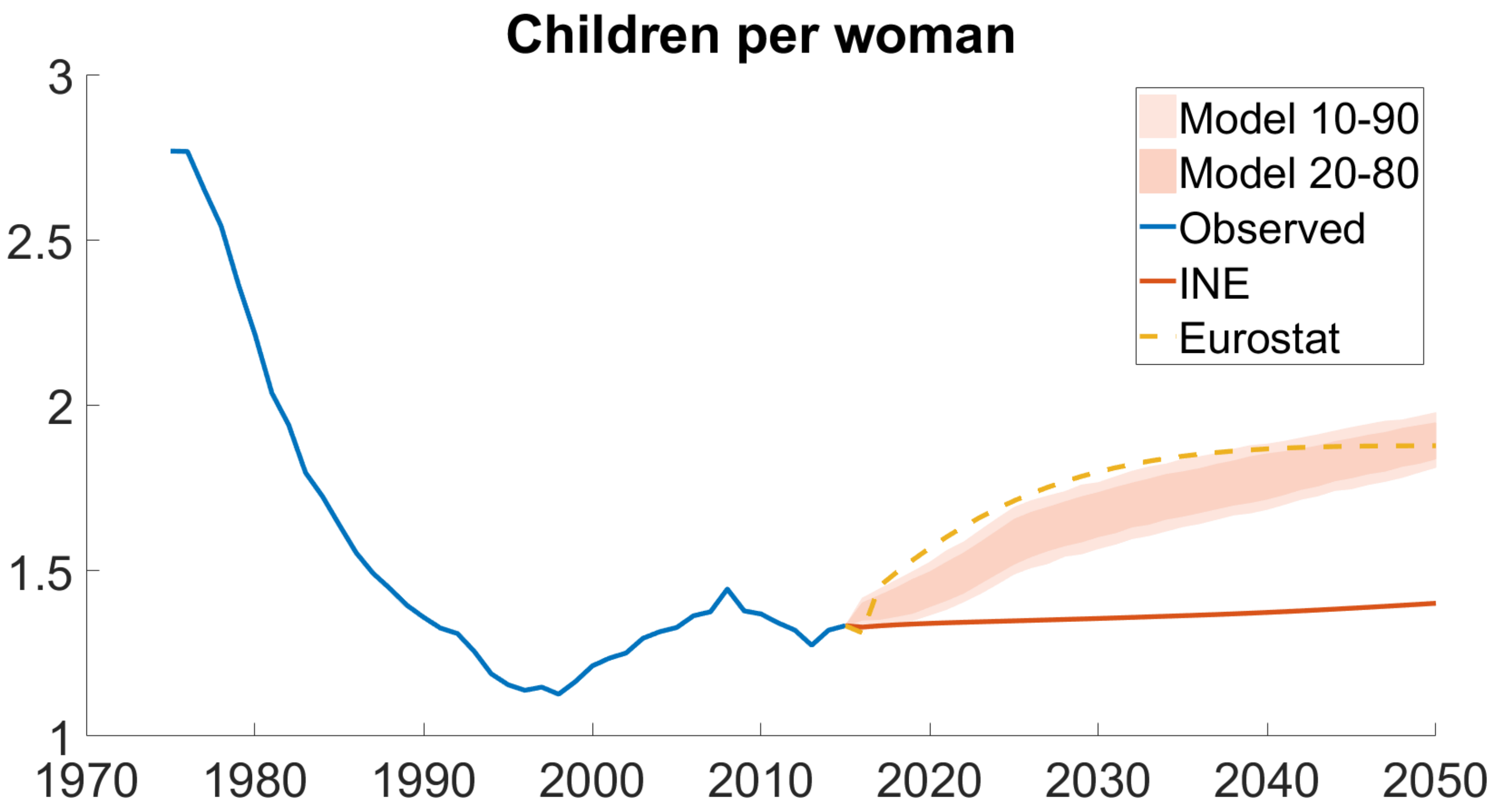
- **Long-term benchmark:** a linear combination of the UN projections. The weights are based on the factor model estimates.

- **Brownian bridge:** conditioning of the Monte Carlo VAR projections to the long-term benchmarks. Poweful (and simple) device that retains stochasticity and tractability.



- **Brownian bridge:** projections of the intensity parameter up to 2100. The terminal value is consistent with the benchmark, given the median values of the other two parameters.

MODELING FERTILITY: Aggregate output



MODELING FERTILITY: a roadmap

Fertility curves: age x year → Parameters: intensity, modal age and dispersion: year → VAR model → Monte Carlo projection → Stochastic scenarios for the curves: age x year

Step 1: **Detailed stochastic** projection using only **national** information

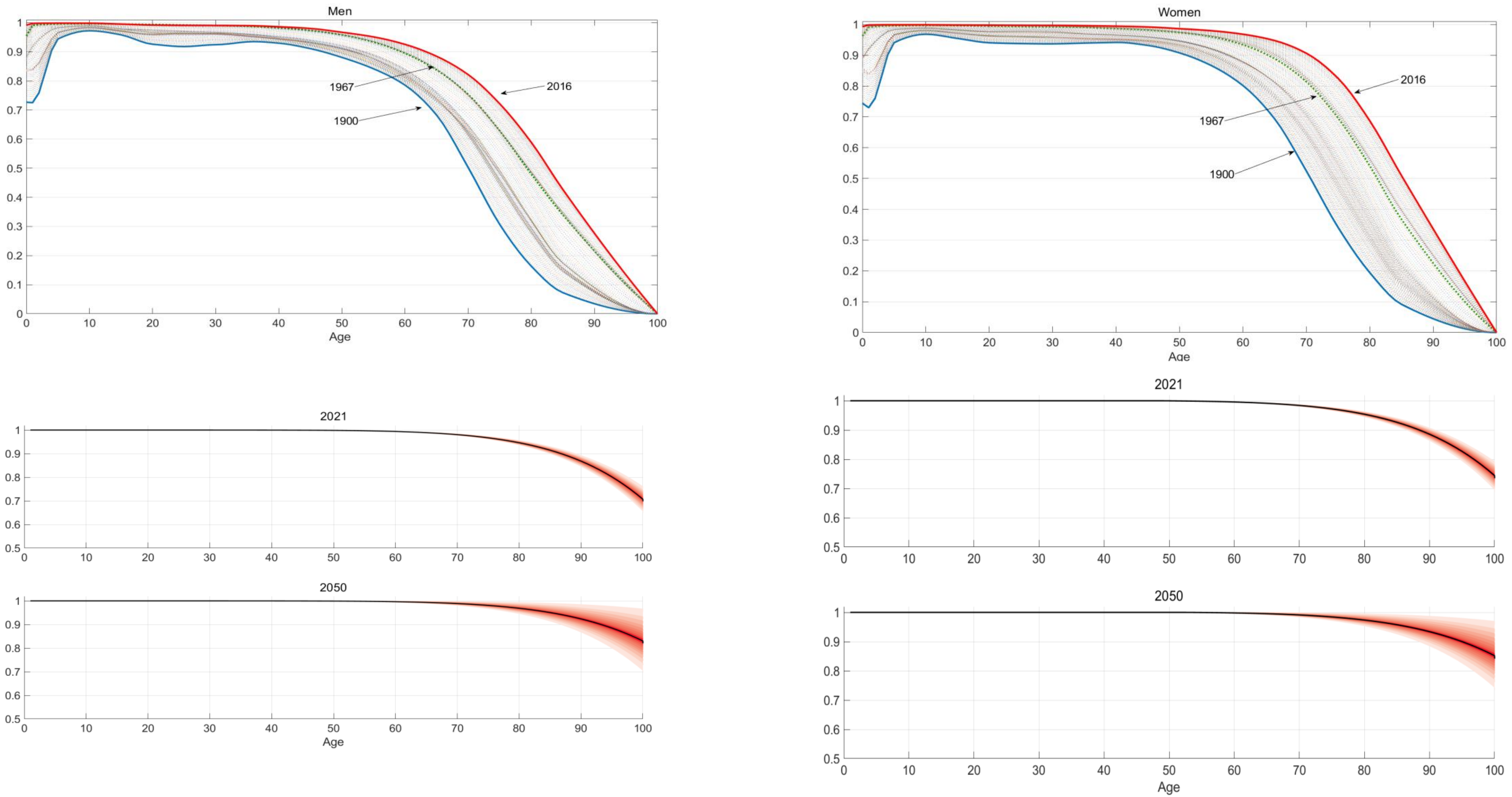
European panel of NCW time series: countries x years → Factor model

- Scoring
- Long-term benchmark

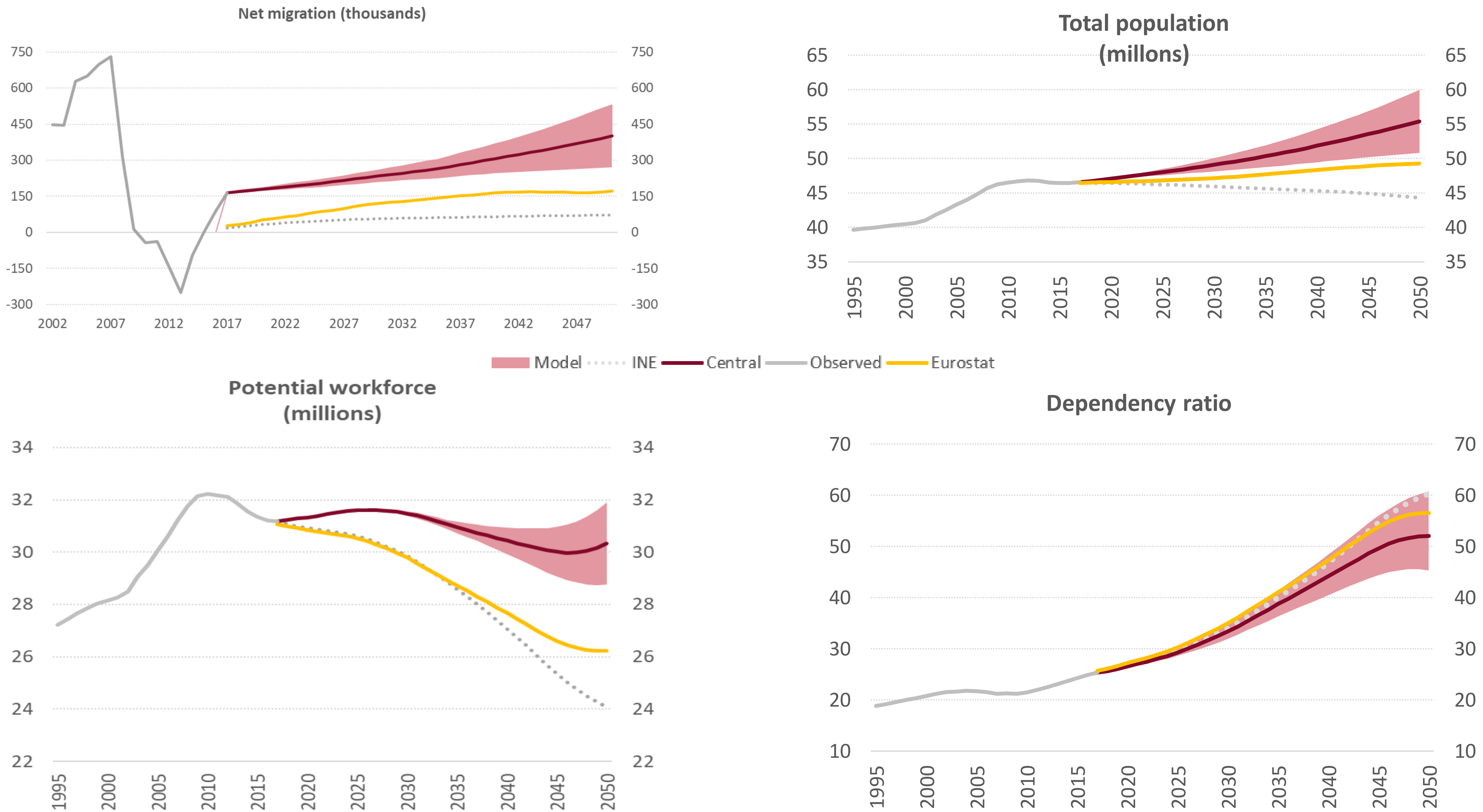
→ Conditioned paths by means of Brownian Bridge device → Conditioned curves → Stochastic population scenarios

Step 2: **Aggregate** conditioning using **international** information

MODELING SURVIVAL



FINAL RECAP: The complete picture



THE ROAD AHEAD

- Improving the merge between domestic and external information: the quest for a unified model that incorporates the Brownian bridge mechanism.

- Improving the Monte Carlo engine: decomposition of uncertainty, conditioned scenarios, etc.

- Structural modeling: e.g. explicit linkage between fertility and labor market conditions. It is a tempting idea that requires careful evaluation because the ideal framework (an Overlapping Generations model) is a beast hard to tame.

Thanks for your attention!

REFERENCES

- AIReF’s Population Projections:

http://www.airef.es/en/news/airef-publishes-its-own-demographic-forecasts/

- AIReF’s Pension Expenditure Projections (in Spanish):

http://www.airef.es/es/gasto-en-pensiones/

APPENDIX: The model among its peers

	NSI (INE)	Eurostat	UN	AIReF
General model	Cohort dynamics			
Gender breakdown	**Yes**			
Age breakdown	Simple ages: [0, 100+]			
Nationality breakdown	Yes	No	No	**Yes**
Fertility	Deterministic	Deterministic assuming long-term international (Europe) convergence	**Stochastic**: Dynamic model for a panel of countries (World, countries in the same demographic phase)	**Stochastic**: Dynamic model conditioned to a panel of European countries
Procedure	Bottom up	Top down	Top down	Bottom up
International information	No	**Yes**	**Yes**	**Yes**
Survival	Deterministic	Deterministic assuming long-term international (Europe) convergence	**Stochastic**: Dynamic model for a panel of countries (World, countries in the same demographic phase)	**Stochastic**: Dynamic model conditioned to a panel of European countries
Procedure	Bottom up	Top down	Top down	Bottom up
International information	No	**Yes**	**Yes**	**Yes**
Migration	Deterministic	Deterministic: convergence to long-term "autharky"	Deterministic: convergence to long-term "autharky"	**Stotchastic**: multilateral gravitatory model
Procedure	Top down	Top down	Top down	Top down
International information	No	Yes	**Yes**	**Yes**